801. The range is the set of output values which it is possible to reach, using inputs from the domain. Since this is a quadratic in x^2, this is most easily found by completing the square.

802. Use the binomial expansion.

803. Draw a force diagram, giving the particle mass m. Label the angle between the weight and normal to the slope θ. Set up NII down the slope, resolving the weight into components.

804. In both parts, use $p = \frac{\text{successful}}{\text{total}}$. The possibility space consists of the ${}^5\text{C}_3 = 10$ ways of choosing the three integers.

805. The terms “increasing” and “decreasing” refer to the value of the first derivative being positive or negative. Write $x\sqrt{x}$ as $x^{\frac{3}{2}}$, then differentiate.

806. (a) Expand to the form $p + q\sqrt{6} = 58 - 12\sqrt{6}$, where p, q are integers in terms of a, b. Then, equate (coefficients of) rational and irrational parts, giving $p = 58$ and $q = -12$.
   (b) One equation has the form $ab = ...$ Make a the subject and substitute in. You'll get a quadratic in b^2. Factorise or use the quadratic formula. Choose the roots such that $a, b \in \mathbb{Z}$, and such that the result is positive.

807. The brute force method here would be to find the distances explicitly by setting up normals to $y = 2x - 3$. This would, however, be a lot more work than is necessary. You can bypass this work by various methods. Two of these are:
   - find the midpoint of OA, and test to see which side of the line it lies on,
   - draw a careful sketch, including $y = 2x - 4$. This passes through the midpoint of OA.

808. It is often useful, in such scenarios, to imagine what would happen is there was no friction. For example, put the car on an ice rank and floor the accelerator. What happens?

809. (a) Read off the x intercept. Set $x = 0$ and solve for any y intercepts.
   (b) Differentiate with respect to y. Switching x and y from their usual roles does not affect the mathematics of the problem: do as you would usually do.
   (c) The curve is a positive parabola (in this case on its side). You found one of the coordinates of the vertex in part (b).

810. Write the sum out longhand, as
$$\frac{1-x}{x} + \text{two other fractions} = \tfrac{1}{2}.$$
Put the fractions over a common denominator and rearrange to make x the subject.

811. Sketch the graph $y = (x - a)(x - b)(x - c)$. This is a positive cubic with three distinct x intercepts. You are then looking for the x values for which y is positive.

812. The roles of x and y have been reversed. Switching x and y is a reflection.

813. Consider the rolls one after another. The first can be anything; the probability of success is 1. So, multiply together two probabilities: of success in the second roll and then of success in the third roll.

814. (a) Use Pythagoras.
   (b) Divide the result of part (a) by $\cos^2\theta$.

815. The gradient of such a parametrically defined line is the ratio of the coefficients of the parameter.

816. Calculate $\sum x$ for the original sample of 100 and for the subset of 10. Hence, calculate $\sum x$ for the remaining 90.

817. Differentiate to find SPs of $y = 20x^2(40 - x^2)$. The graph has three: you'll need to distinguish between the (global) maxima and the (local) minimum.
Alternatively, since this quartic is a quadratic in x^2, complete the square.

818. In each case, just follow the actions of the functions on the inputs. Remember that $\text{gf}(a)$ means “apply f to a first, then apply g”, because f is closest to the input a.

819. Consider a journey once around the perimeter. At each vertex, the change in direction is given by the exterior angle. Be careful not to assume the special case of a *regular* polygon, which is not mentioned in the question.

820. (a) Use $u_{n+1} = u_n + d$, which holds by definition of the common difference d of an AP.
   (b) Substitute part (a) into the formula for w_n, and rearrange to the form $w_n = su_n + t$. This must be an AP: explain why by working out the common difference.

821. Horizontally, there is no velocity, so the distance remains constant. Vertically, set up two *suvat* equations for the positions, and find the difference between them.

822. Factorise the second quadratic, and compare the roots. Also, compare the leading coefficients.

823. Subtract the unshaded regions from the total.

824. Consider the discriminant of the quadratic.

825. These are parts of the $y = |x|$ graph. Remember: the square root function and the squaring function do not perfectly undo one another: the symbol $\sqrt{}$ means "the *positive* square root of...".

826. Use the binomial expansion. Expand $(2x + 3)^4$ first, then write down the expansion of $(2x - 3)^4$ without further calculation. Odd powers of x will cancel, and you should end up with a quadratic in x^2.

827. Remember that an integral is a continuous sum (the integral symbol is a big curly S for *Sum*) of the values of a function. This includes the sign of the function.

828. Divide both sides by $\cos x$ and use $\tan x \equiv \dfrac{\sin x}{\cos x}$.

829. Add $4xy$ to both sides first.

830. Consider the value of the middle angle.

831. Draw the path of the relevant vertex under the next rotation. This will take it back to the x axis. The third rotation leaves the vertex where it is. These three rotations produce one full cycle.

832. Assume, for a contradiction, that three distinct points on a parabola $y = ax^2 + bx + c$ are collinear. Let the line have equation $y = px + q$. Consider the number of roots of the equation for intersections.

833. (a) "Under canopy" means that tension is being exerted by cords attached to the canopy (the main part of a parachute). This should be modelled as a single tension acting vertically.
   (b) Use NII vertically.
   (c) Your new force diagram should now include a reaction acting upwards on the parachutist.
   (d) Calculate the average acceleration during the 0.4 seconds of landing. Use this in NII.

834. This is a quadrant-like region.

835. Consider the odd and even cases separately.

836. Translate into algebra and integrate.

837. Parabola P_2 is $y = (x - k)^2$.

838. Multiply out the proposed integral. Differentiate it term by term and show that it doesn't come to $x(x^2 + 1)$.

839. This is a geometric series. Use the standard sum
$$S_n = \frac{a(r^n - 1)}{r - 1}.$$
Then convert from grams into tonnes.

840. Multiply up by the denominators and rearrange to make x the subject. Then consider the domain of definition of the square root function.

841. The possibility space is contained in a 5×5 grid. However, because the integers are distinct, only 20 of the 25 squares are valid outcomes.

842. (a) i. The gradient of the hypotenuse is $-\frac{b}{a}$. Use $(0, b)$ to find the equation.
   ii. Hypotenuse and radius are perpendicular.
   (b) Solve the equations from (a) simultaneously.
   (c) Find $|OP|^2$ using Pythagoras.

843. Consider, using the discriminant $\Delta = b^2 - 4ac$, whether the denominator can equal zero.

844. These are two straight lines, passing through the points (p, q) and $(-p, -q)$.

845. The factor theorem, over $\mathbb{R}$, cannot be used here, because the quadratic $1 + x^2$ has no real roots. Factorise explicitly, taking out $(1 + x^2)$.

846. The shaded region is a rhombus consisting of two equilateral triangles. Show that the height of the hexagon is $\sqrt{3}$, and find the height of the two small isosceles triangles at the top and bottom. Subtract to find the side length of the equilateral triangles that make up the shaded region.

847. Whichever directions the vectors $\mathbf{a}$ and $\mathbf{b}$ are in, you can rotate/reflect the problem to align them with x and y axes. So, you can assume, without loss of generality, that $\mathbf{a}$ and $\mathbf{b}$ are the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$. Having made this assumption, work with gradients.

848. Split the fraction up before integrating.

849. The angle of projection which attains maximum range over flat ground is 45°.

850. The shortest path from a point to a line is along the normal.

851. The implication is only valid if the only root of the second factor is $x = 0$.

852. This is a quartic in $\sin x$. Factorise it. One of the factors has no real roots. The other has two.

853. In each case, consider the numbers that serve as inputs to the function g.

854. Give a counterexample: two linear simultaneous equations in x and y which are satisfied either by no points (x, y) or by infinitely many.

855. Use standard trig values.

856. Multiplication by $x^2 + y^2$ adds a point back in, for which the original equation is undefined.

857. Consider the restriction of the possibility space brought about by the information given. The set $\{1, 2, 3, 4, 5, 6\}$ is restricted to $\{2, 4, 6\}$.

858. Differentiate $y = \frac{1}{2}x^2$ and substitute $x = a$. Relate $\tan\theta$ to the gradient of the tangent.

859. (a) The double root implies a point of tangency with the x axis.
   (b) Multiply out the brackets to give
   $$\int_0^a 12x^3 - 24ax^2 + 12a^2x\,dx.$$
   Perform this integral.

860. To visualise these, sketch a number line. Square brackets are inclusive of the boundary; round brackets are exclusive of it. One of the answers is the empty set $\varnothing$.

861. Sketch the scenario.

862. Solve the boundary equation $t^3 - 4t = 0$ and sketch the cubic $y = t^3 - 4t$.

863. Put the fractions over a common denominator of $1 - x^2$. Then rearrange to make x^2 the subject, before taking the square root.

864. This is equivalent to saying that $(x + 1)$ is not a factor of the cubic.

865. (a) Subtract the angles from 180°. You don't have to do this exactly, as the question only asks "find", not "determine". So, just find a value in degrees to 1dp.
   (b) In a triangle, the shortest side is opposite the smallest angle. Use the sine rule.

866. The curve is an ellipse centred on the origin.

867. Consider the following fact: two vectors are scalar multiples of one another if and only if they are parallel.

868. The boundary equation is a quadratic. Solve it. Then sketch the graphs $y = \text{LHS}$ and $y = \text{RHS}$.

869. Solve for Δ.

870. In both (a) and (b), the ratio of the magnitudes of numerator and denominator approaches 1. The only difference is the sign. If in doubt, just plug some large values into your calculator.

871. Since $(9, 3)$ is in the positive quadrant, you can use $y = \sqrt{x}$ rather than $x = y^2$. Differentiate to find the gradient of the tangent, then the gradient and equation of the normal. Set $y = 0$ and solve.

872. (a) Use *suvat*.
   (b) Consider the speed of the up projectile when it returns to its initial position.

873. $\triangle AXC$ is right-angled, congruent to $\triangle ABC$.

874. Calculate the second derivative and substitute.

875. (a) In each, use $p = \frac{\text{successful}}{\text{total}}$.
   (b) Divide the top and bottom by k, before taking the limit.

876. (a) Set up the cosine rule for d_1^2 and d_2^2. Use the exact trig values for $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ and simplify.
   (b) Counting right-angled triangles, the area of a rhombus is given by $A = \frac{1}{2}d_1d_2$.

877. Consider the roots of each factor.

878. Solve as a pair of linear equations in x^2 and y^2.

879. (a) Set up and solve an equation in k.
   (b) The result follows directly from an index law.

880. Quote the formulae for A and l in terms of the radius r and θ, and eliminate r.

881. Consider a common factor.

882. Reciprocate according to the definition
   $$\sec\theta \equiv \frac{1}{\cos\theta}.$$
   Then rationalise the denominator.

883. Find a root and thereby a factor by inspection or numerical methods. Then factorise fully.

884. (a) For the LHS to be negative, exactly one of its factors must be negative.
   (b) Concentric circles have the same centre. In this case, the centre is the origin.

885. The acceleration under gravity on a smooth slope of angle θ is $g \sin\theta$. This is a fact worth knowing. The component of weight is $mg\sin\theta$.

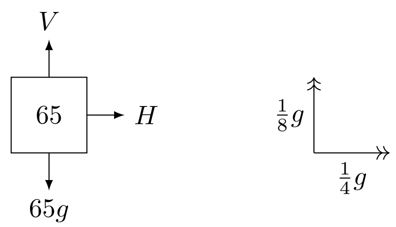
886. Put the fractions over a common denominator.

887. (a) Use $y - y_1 = m(x - x_1)$ or $y = mx + c$.
   (b) Eliminate y from the equation of the curve and your equation in (a).
   (c) The tangent line has exactly one intersection with the quadratic. So, set $\Delta = 0$ and solve.
   (d) Solve simultaneously.

888. Express the two rational numbers as quotients of integers: $\frac{a}{b}$ and $\frac{c}{d}$. Add them explicitly.

889. Consider the value $x = 0$.

890. Split the contact force into its components. Giving these magnitudes V and H, the force diagram for the passenger is



Use NII horizontally and vertically, then combine the two with Pythagoras.

891. Draw the possibility space as a six-by-six grid, and count successful outcomes.

892. (a) Factorise fully, taking out $(x - 2)^2$.
   (b) Differentiate the original form.
   (c) Substitute in the values from (a).
   (d) Consider that a double root corresponds to a point of tangency with the x axis.

893. Consider the greatest and smallest values of the magnitude of F, as determined by the directions of the 50 and 60 Newton forces.

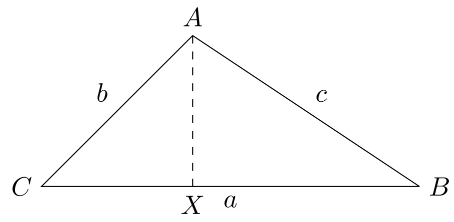
894. Translate this into algebra, and integrate.

895. Consider the straight sections and curved sections separately. Together, the curved sections form the circumference of a circle.

896. Find a counterexample: start with $n = 1$ and work upwards, looking for either $n! + 1$ or $n! - 1$ being non-prime.

897. Multiply out first.

898. Use the following setup:



Find $|AX|$ and $|BX|$ in terms of a, b and C. Then use Pythagoras on triangle ABX.

899. (a) Set up simultaneous equations. In one, use a duration of $t - 2$ to express the fact "sets off two second later".
   (b) Solve for t, then use *suvat*. Then be careful to answer the correct question.

900. (a) Use $p = \frac{\text{successful}}{\text{total}}$.
   (b) Test for independence with

$$\mathbb{P}(X \in B) \times \mathbb{P}(X \in C) = \mathbb{P}(X \in B \cap C).$$

—— END OF 9TH HUNDRED ——